THE PROBLEM OF THE BURNING OF AN ELECTRIC ARC IN A STREAM OF GAS

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A one-dimensional heat-conduction equation is analyzed for the positive column of an arc discharge in a lateral gas flow ($V \perp j$). Two discharge burning regimes are found for the same parameters (E and V). The critical gas flow rate at which disruption of the burning occurs is determined. The volt-ampere characteristics of the discharge are constructed.

In connection with the development of new branches of engineering (electric-arc heaters, engines, motors, etc.) the attention of researchers is drawn

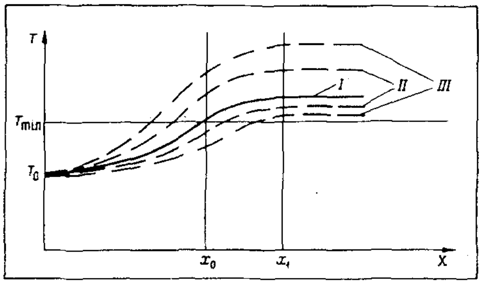

Fig. 1. Temperature distribution in flow direction for various gas velocities. I) $\alpha = \alpha_{max}$; II) $\alpha = \alpha_1$; III) $\alpha = \alpha_2$ ($\alpha_1 > \alpha_2$).

to problems involving the burning of an electric arc in a gas stream [1, 2]. Particular interest is exhibited in the case of lateral streamlining of an arc ($V \perp j$).

The problem of the burning of an electric arc is examined in [1] on the basis of the equation of energy balance, with consideration of the strong nonlinear dependence on the temperature of Joule heating. An attempt is made here to clarify the spatial distribution (in a one-dimensional approximation) of temperature and thus to find the volt-ampere characteristic of the arc for various values of gas-flow velocity and temperature. As in [1], here we will examine the following region of temperature and velocity values: V, up to 100 m/sec; $T_0 = 300–3000°$ K.

The processes occurring at the electrodes (heat transfer, material vaporization, disruption of quasi neutrality) are of great significance for arc discharge. However, for long arcs with great current strength the effect of electrode processes on the volt-ampere characteristics is small [3]. As in [1], it is therefore also of interest to ascertain the effect of the flow on the characteristics of the positive arc-discharge column.

It is well known from experimental work [1, 2] that with spatially bounded electrodes in the case of lateral streamlining the arc begins to bend and the discharge combustion pattern becomes substantially different from one-dimensional. However, as before, the central portion of the bend may be treated as one-dimensional. Moreover, streamlining is possible in the presence of a stabilizing magnetic field, which also makes possible the one-dimensional approximation of arc burning without consideration of boundary effects.

The energy balance for the positive column of a free-burning electric arc in a gas stream for the steady case can be written as follows:

$$\sigma E^2 + \operatorname{div}(\varkappa \operatorname{grad} T) - \rho c_p (\mathbf{V} \cdot \operatorname{grad} T) = 0. \quad (1)$$

In the solution of the problem we will treat the quantities $\varkappa$, c_p, and ρV as constant and given. The constancy of ρV requires no justification since it is a consequence of the law of the conservation of mass.

The constancy of $\varkappa$ and c_p is an approximation which is valid because the effect of a supply of heat is examined in the problem for the electrical conductivity (σ) as a function of temperature according to the law $\sigma \approx \exp(-e\varphi_i/2kT)$. And since $\varkappa$ and c_p are considerably weaker functions of T than the function $\sigma = f(T)$, we can expect that consideration of their temperature relationship will introduce no significant variations in the distribution $T = T(x)$. The qualitative picture is apparently preserved, although there may be quantitative variations. In that case, for the one-dimensional case ($V = V_x = V$) we obtain

$$T'' - \frac{V \rho c_p}{\varkappa} T' = -\frac{\sigma E^2}{\varkappa} \quad (2)$$

or

$$T'' - \alpha T' = -\frac{\sigma E^2}{\varkappa}, \quad (3)$$

where

$$\alpha \equiv \frac{c_p \rho V}{\varkappa} = \text{const.} \quad (3a)$$

The field E for the positive column may be regarded as homogeneous, and in the one-dimensional case, neglecting the boundary effects, as independent of the coordinates x within the limits of electrode length.

The field E can consequently be written as

$$E = \begin{cases} 0 \text{ when } x < x_0 \text{ and } x > x_1, \\ E \text{ when } x_0 \leqslant x \leqslant x_1. \end{cases} \quad (4)$$

Thus, if the initial gas-flow temperature is assumed to be equal to T_0, the problem reduces to the solution of the nonlinear equation (3) for the following condition:

$$T|_{x\to-\infty} \to T_0. \tag{5}$$

We will divide the entire region of variation in x into three intervals:

$$\underset{-\infty < x < x_0,}{\text{I}} \qquad \underset{x_0 \leqslant x \leqslant x_1,}{\text{II}} \qquad \underset{x_1 < x < +\infty.}{\text{III}}$$

In the first and third intervals E = 0 and (3) becomes a linear equation with constant coefficients. The solution of (3) in these intervals can be represented in the following form:

$$T_1(x) = C_1 \exp\alpha x + C_2 \quad (-\infty;\ x_0),$$
$$T_3(x) = C_5 \exp\alpha x + C_6 \quad (x_1;\ +\infty). \tag{6}$$

The constant C_5 must be set equal to 0, since $\alpha > 0$. In the interval $x_0 \le x \le x_1$ Eq. (3) is nonlinear because of the function $\sigma(T)$. With simple transformations it can be reduced to an integral equation of the form

$$T_2(x) = C_4 + C_3 \exp\alpha x - -\frac{1}{\alpha\varkappa}\int_{x_0}^{x} \sigma E^2 [\exp\alpha(x-t) - 1]\,dt. \tag{7}$$

To determine the constants we will use the condition (5) and the condition of joining the solutions at the points x_0 and x_1:

$$T_1(x_0) = T_2(x_0),\ \left.\frac{dT_1}{dx}\right|_{x=x_0} = \left.\frac{dT_2}{dx}\right|_{x=x_0},$$
$$T_2(x_1) = T_3(x_1),\ \left.\frac{dT_2}{dx}\right|_{x=x_1} = \left.\frac{dT_3}{dx}\right|_{x=x_1}. \tag{8}$$

Proceeding from the formulas of (8), we derive the following expressions for the constants:

$$C_2 = C_4 = T_0;\quad C_1 = C_3 = -\frac{1}{\alpha\varkappa}\int_{x_0}^{x_1} \sigma E^2 \exp(-\alpha t)\,dt,$$
$$C_6 = T_0 + \frac{\exp\alpha x_1}{\alpha\varkappa}\int_{x_0}^{x_1} \sigma E^2 \exp(-\alpha t)\,dt - -\frac{1}{\alpha\varkappa}\int_{x_0}^{x_1} \sigma E^2 [\exp\alpha(x-t) - 1]\,dt. \tag{9}$$

Consequently, for the entire flow region $(-\infty;\ +\infty)$ the expression

$$T(x) = T_0 + +\frac{1}{\alpha\varkappa}\int_{x}^{x_1} \sigma E^2 \exp\alpha(x-t)\,dt - \frac{1}{\alpha\varkappa}\int_{x_0}^{x} \sigma E^2 dt \tag{10}$$

turns (3) into an identity.

We see from (10) that in the region $x < x_0$ temperature increases exponentially while in the region

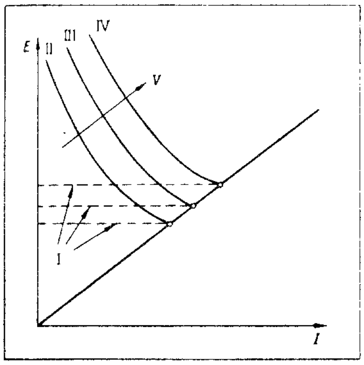

Fig. 2. Volt-ampere characteristics of discharge for various gas velocities. I) E = $E_{\text{disruption}}$; II) V_1; III) V_2; IV) V_3 ($V_1 < V_2 < V_3$).

$x_0 \le x \le x_1$ the rise in temperature is retarded, since for $x = x_1$, grad T = 0 at the boundary. In the region $x \ge x_1$ the temperature becomes constant, which follows directly from the formulation of the problem (we neglect heat removal).

Strictly speaking, (10) yields no solution for (3), but represents an integral equation for the function T(x). In the general case it can be solved by the method of successive approximations.

We know that the electrical conductivity of an ionized gas is a function of temperature primarily according to the law

$$\sigma \sim \exp\left(-\frac{e\varphi_i}{2kT}\right). \tag{11}$$

Hence we can see that up to some temperature, $T_{\min}\sigma$ will be a very small quantity, while for large temperatures it will tend toward a constant value (to a completely ionized gas).

Therefore, just as in (4), an acceptable assumption will be the representation of the temperature function in the form of a step function:

$$\sigma = 0,\quad \text{when } T < T_{\min},$$
$$\sigma = \sigma_0,\quad \text{when } T \geqslant T_{\min}. \tag{12}$$

Since we assume that the free stream is not ionized, a natural limitation for $T_{\min}$ will be $T_0 < T_{\min}$. For this approximation a single solution is obvious: $T = T_0$ for all x. The remaining two solutions are derived from an examination of the two possible cases in which

$$x_{\min} < x_0, \tag{13a}$$
$$x_0 < x_{\min} < x_1, \tag{13b}$$

where $x_{\min}$ is defined by the condition $T(x_{\min}) = T_{\min}$.

Let us examine the conditions for the existence of solutions for both cases. Having used (10) and (12), we find the expressions for $x_{\min}$:

$$x_{\min} = \frac{1}{\alpha} \ln \frac{(T_{\min} - T_0)\alpha^2 \varkappa}{\sigma_0 E^2 [\exp(-\alpha x_0) - \exp(-\alpha x_1)]}, \quad (14a)$$

$$x_{\min} = x_1 + \frac{1}{\alpha} \ln \left[1 - \frac{(T_{\min} - T_0)\alpha^2 \varkappa}{\sigma_0 E^2}\right]. \quad (14b)$$

In examining (13) and (14) we found that for both cases the condition for the existence of a solution is defined by the inequality

$$\frac{(T_{\min} - T_0)\alpha^2 \varkappa}{\sigma_0 E^2} \leqslant 1 - \exp \alpha (x_0 - x_1). \quad (15)$$

Formula (15) demonstrates that for a given electrode width ($l = x_1 - x_0$) at any flow rate there exists a

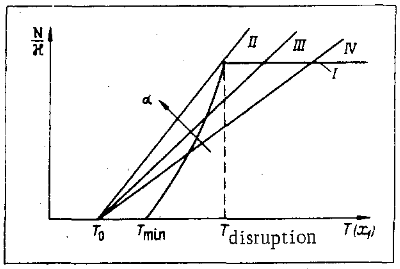

Fig. 3. Graphical solution of Eq. (25). I) $f[T(x_1)]$; II) $\alpha_1 [T(x_1) - T_0]$; III) $\alpha_2 [T(x_1) - T_0]$; IV) $\alpha_3 [T(x_1) - T_0]$ ($\alpha_1 > \alpha_2 > \alpha_3$).

minimum value for the electric field $E_{\min} = E_{\text{disruption}}$ below which the arc will not burn. Conversely, for any field value there exists a maximum flow-rate value $V_{\max} = V_{\text{disruption}}$, beginning from which the discharge is extinguished (absence of solution). The value of the parameters at which disruption of burning takes place can be derived from (15). We will present the formulas for T(x) in the region of existence ($x_0 \leq x \leq x_1$) for the electric field, these formulas having been derived on substitution of (12) into (10):

$$T(x) = T_0 + \frac{\sigma_0 E^2}{\alpha^2 \varkappa}[1 - \exp \alpha (x - x_1)] + \frac{\sigma_0 E^2}{\alpha \varkappa}(x - x_0), \quad (16a)$$

$$\left.\begin{aligned} T(x) &= T_0 + \frac{\sigma_0 E^2}{\alpha^2 \varkappa} \times \exp \alpha x \left[\exp(-\alpha x_{\min}) - \exp(-\alpha x_1)\right] \Big|_{x_0 < x < x_{\min}} \\ T(x) &= T_0 + \frac{\sigma_0 E^2}{\alpha^2 \varkappa}\left[1 - \exp \alpha (x - x_1)\right] + \frac{\sigma_0 E^2}{\alpha \varkappa}(x - x_{\min}) \Big|_{x_{\min} < x < x_1} \end{aligned}\right\}. \quad (16b)$$

The behavior of the functions derived from these formulas is shown in Fig. 1. We see that for the identical value of the parameters E and α there are two solutions: one solution when the temperature $T_{\min}$ is attained outside the region of the field, and the other, when it is attained within the limits of the field.

We see from Fig. 1 that for solutions corresponding to (13a) and (15), the temperature drops throughout the entire space as the flow rate (or α) increases. However, with (13b) the process is the opposite. On attaining the flow rate transforming (15) into an equality, both solutions coincide. This corresponds to the situation in which both curves merge and intersect the left-hand boundary of the field at the temperature $T = T_{\min}$ (i. e., $x_{\min} = x_0$). The solution for this case corresponds to the disruption of the discharge burning in the flow, because at great flow rates the temperature of the gas is transformed into an identical constant $T = T_0$ over the entire space. Thus, for the identical values of E and α there exist two different temperature curves. This leads to a situation in which, for fixed E and V, there will exist two stream values.

Let us construct the volt-ampere characteristics of the discharge for a fixed value of the flow rate (α = const). The total current is equal to

$$I = \int j dS \quad (17)$$

for the one-dimensional case and for an electrode of unit width across the flow

$$I = \int j dx = \int \sigma E dx = \sigma_0 E \int dx. \quad (18)$$

The last integral is taken with respect to the region in which σE is different from 0. Then for (13a)

$$I = \sigma_0 E (x_1 - x_0), \quad (19a)$$

and for (13b)

$$I = \sigma_0 E (x_1 - x_{\min}). \quad (19b)$$

Having substituted the expression for $x_{\min}$ from (14b) into (19b) we obtain

$$I = -\frac{\sigma_0 E}{\alpha} \ln \left[1 - \frac{(T_{\min} - T_0)\alpha^2 \varkappa}{\sigma_0 E^2}\right]. \quad (20)$$

From (19a) and (20) it is possible to construct the volt-ampere characteristics, considering that E varies from $E = E_{\text{disruption}}$ to $E \to \infty$ (see Fig. 2). It is easy to see that (19a) corresponds to a growing characteristic. However, (20) yields a dropping characteristic which is easy to prove by taking the derivative $(\partial I/\partial E)_{\alpha=\text{const}}$. For large E the current from (20) approaches zero according to the law

$$I \to \frac{(T_{\min} - T_0)\alpha \varkappa}{E}. \quad (21)$$

The volt-ampere characteristics for large values of the flow rate are situated higher, with the growing portions merging into a single line (19a). This is a result of the fact that we approximated the function $\sigma(T)$ = const when $T > T_{\min}$.

We now note the relationship between the solution of (3) with the results derived in [1].

We integrate (3) over the entire space $(-\infty; +\infty)$

$$\int_{-\infty}^{+\infty} (T'' - \alpha T')\, dx = -\frac{N}{\varkappa} = -\frac{E^2}{\varkappa} \int_{-\infty}^{+\infty} \sigma\, dx. \qquad (22)$$

The left-hand part is equal to (the boundary conditions having been used)

$$\int_{-\infty}^{+\infty} (T'' - \alpha T')\, dx = T'(\infty) - T'(-\infty) - \alpha T(\infty) + $$

$$+ \alpha T(-\infty) = -\alpha [T(x_1) - T_0]. \qquad (23)$$

The integrand in the right-hand part of (22) is different from zero in the region in which $T \geq T_{min}$.

Having used (14a) and (14b) we can present the integral in the right-hand part of (22) in the form

$$-\frac{N}{\varkappa} = -f[T(x_1)]. \qquad (24)$$

Having equated (23) and (24), we derive the equation for the energy balance in integral form

$$\alpha [T(x_1) - T_0] = f[T(x_1)]. \qquad (25)$$

Equation (25) is analogous to the equation used in [1]. The left-hand part of (24) represents (correct to the factor $1/\varkappa$) convective removal of energy from the arc per unit time and the right-hand part is the contributed electrical power.

Figure 3 shows the form of the function $f[T(x_1)]$. The intersection points of the curve $f[T(x_1)]$ with the straight lines $\alpha[T(x_1) - T_0]$ yield the values of the finite gas temperature.

As in [1], three possible states have been obtained. However, it should be noted that the form of the function f can be derived only after solution of the differential equation (3).

NOTATION

Here T is the gas temperature; T_0 is the initial gas temperature; T_{min} is the gas temperature at which electrical conductivity is not zero; E is the electrical field voltage; V is the gas velocity; ρ is the gas density; c_p is the specific heat of the gas; $\varkappa$ is the thermal conductivity; σ is the electrical conductivity; N is the electrical power; φ_i is the ionization potential; e is the electron charge; I is the electric current; j is the electric current density.

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